A posteriori Parameter Selection for Local Regularization of Nonlinear Volterra Equations of Hammerstein Type

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Abstract

Solutions of linear and nonlinear inverse problems, particularly those with special structure or for which non-smooth solutions are expected, can be effectively reconstructed using local regularization methods. For Volterra problems, the method retains the causal structure of the original problem, in contrast to classical regularization methods, and leads to fast sequential numerical algorithms to solve the inverse problem.

Local regularization was originally applied to the nonlinear Volterra problem of Hammerstein type in Lamm and Dai (2005), where the localized approach led to a two-step solution method; one linear step followed by one fully nonlinear step. The method was improved upon in Brooks, Lamm, and Luo (2010), where advantage was taken of the local nature of the method in order to implement local regularization and an effective linearization strategy all at once. The new method retains the causal structure of the original Volterra problem, still allows for fast, sequential numerical solution, however its numerical implementation no longer involves numerous inversions of a nonlinear function as required previously.

We present convergence results for this new method for the finitely-smoothing convolution problem given noisy data. The convergence is achieved with an *a posteriori* choice of the regularization parameter using an adapted version of a newly defined modified discrepancy principle in Brooks (2007). We conclude with a brief discussion of numerical implementation and examples.

This is joint work with Patricia K. Lamm at Michigan State University.

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